1.   Consider a magnetic circuit shown in the following picture.
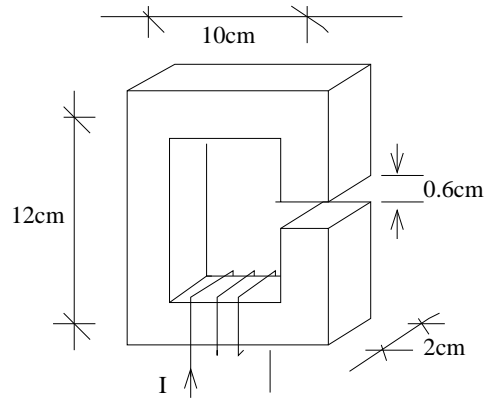

The current I is 10A, the coil has 2000 turns, all branches have the same cross-sectional area of 2cm^2 and the core is iron μ_r of which is 1500. Find the reluctance $\mathcal{R}$, the magnetomotive force $\mathcal{F}$ and the magnetic flux Ψ, firstly for the core and then again for the air gap.

Solution.

a.
$$\mathcal{F} = NI = 2000(10) = 2 \times 10^4 \quad \text{A} \cdot \text{turns}$$

$$\#$$

$$\mathcal{R} = \frac{l}{\mu S} = \frac{0.24 + 0.2 - 0.006}{1500(4\pi)10^{-7}(2 \times 10^{-4})}$$
$$= 1.15 \times 10^6 \quad \text{A} \cdot \text{turns} \cdot \text{Wb}^{-1}$$

$$\#$$

$$\mathcal{F} = \Psi \mathcal{R}$$
$$2 \times 10^4 = 1.74 \times 10^{-2} = 17.4 \quad \text{mWb}$$

$$\#$$

b.
$$\mathcal{R} = \frac{0.006}{4\pi 10^{-7}(2 \times 10^{-4})} = 2.4 \times 10^7 \quad \text{A} \cdot \text{turn} \cdot \text{Wb}^{-1}$$

$$\#$$

$$\mathcal{F} = 2 \times 10^4 \quad \text{A} \cdot \text{turns}$$

$$\#$$

$$\Psi = \frac{2 \times 10^4}{2.4 \times 10^7} = 8.3 \times 10^{-4} = 0.83 \quad \text{mWb}$$

$$\#$$

2.   In a cube of size a, assuming k_0 constant, suppose we know the following holds.

$$\mathbf{M} = \frac{k_0}{a}(-y\mathbf{a}_x + x\mathbf{a}_y).$$

Find the magnetisation volume current density $\mathbf{J}_b$.

Solution.

$$\begin{aligned}
\mathbf{J}_b &= \nabla \times \mathbf{M} \\
&= \left[\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}\right]\mathbf{a}_z \\
&= \left(\frac{k_0}{2} + \frac{kK_0}{2}\right)\mathbf{a}_z \\
&= \frac{2k_0}{a}\mathbf{a}_z \quad \text{A} \cdot \text{m}^{-2}
\end{aligned}$$

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3.   Consider a material μ of which is $6.5\mu_0$. Suppose the magnetic fields is $\mathbf{H} = 10\mathbf{a}_x + 25\mathbf{a}_y - 40\mathbf{a}_z \mathrm{A} \cdot \mathrm{m}^{-1}$. Find the magnetic susceptibility χ_m of this material, the magnetic flux density $\mathbf{B}$, the magnetization $\mathbf{M}$, and the magnetic energy density w_m.

   Solution.

a.
$$\frac{\mu}{\mu_0} = 1 + \chi_m$$
$$\chi_m = \frac{6.5\mu_0}{\mu_0} - 1 = 5.5$$

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b.
$$\mathbf{B} = \mu\mathbf{H}$$
$$= 6.5\mu_0(10\mathbf{a}_x + 25\mathbf{a}_y - 40\mathbf{a}_z)$$
$$= \mu_0(65, 162.5, -260)$$
$$= (81.6, 204.2, 326.7) \quad \mu\mathrm{A} \cdot \mathrm{m}^{-1}$$

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c.
$$\mathbf{M} = \chi_m\mathbf{H}$$
$$= 5.5(10\mathbf{a}_x + 25\mathbf{a}_y - 40\mathbf{a}_z)$$
$$= (55, 137.5, -220) \quad \mathrm{A} \cdot \mathrm{m}^{-1}$$

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d.
$$w_m = \lim_{\Delta v \to 0} \frac{\Delta w_m}{\Delta v} = \frac{1}{2}\mu H^2$$

But $H = \sqrt{10^2 + 25^2 + 40^2} = 48.2$, so $H^2 = 2325$. Therefore,

$$w_m = \frac{1}{2}6.5(4\pi10^{-7})2325$$
$$= 9.5 \times 10^{-3}$$
$$= 9.50 \quad \mathrm{mJ} \cdot \mathrm{m}^{-3}$$

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